3-1 Derivative of a Function

Learning Objectives:

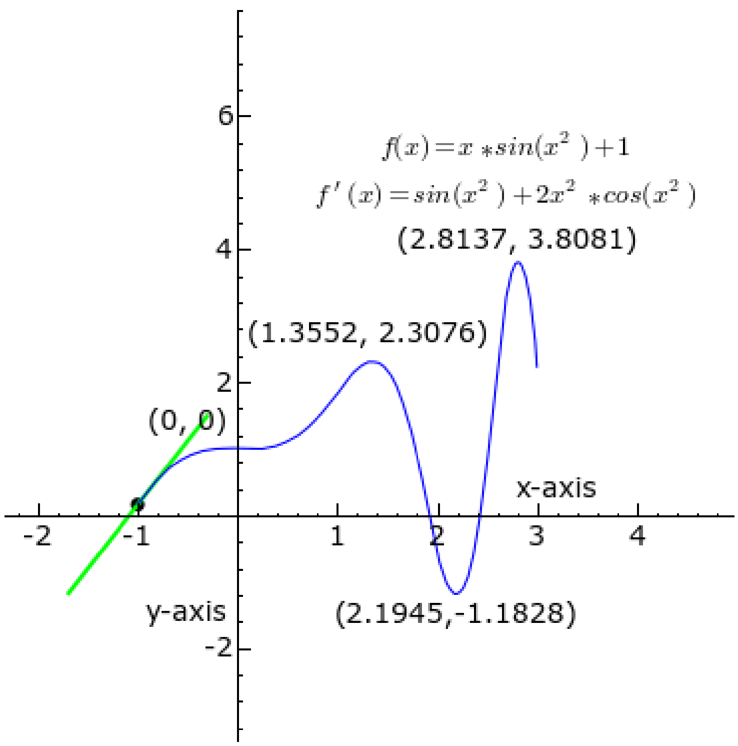
I can calculate the derivative of a function using the definition of derivative.

I can express derivatives using different notations.

I can graph the derivative of a function.

I can interpret derivatives as rates of change and can identify the units.

I can write the equation of the tangent line to a curve.



A **Tangent Line** is a line that touches the graph of a curve exactly once. The **Derivative** is the slope of this tangent line.

Definition of Derivative

The derivative of a function f(x) with respect to the variable x is the function f ‘ (x) whose value at x is given by:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

if the limit exists.

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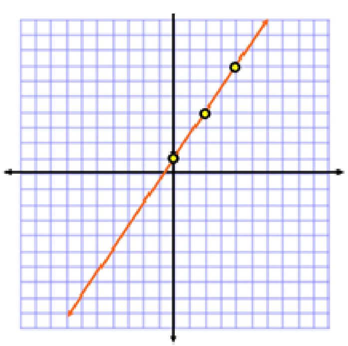
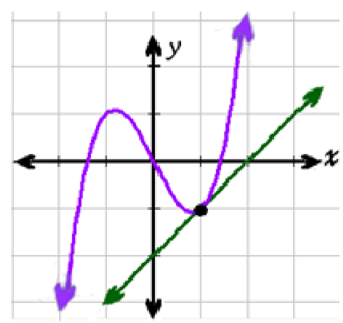
Derivative ⇄ Slope ⇄ Rate of Change

The only difference between the derivative in Calculus and the slope in Algebra is that in Algebra the slope is fixed at a constant value everywhere in the function whereas in Calculus the slope is ever changing throughout the domain of the function.

Algebra



Calculus



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Ex1. Use the definition to find the value of the derivative

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1.) $f(x) = x^2$

$$f' = \lim_{h \to 0} \frac{[(x+h)^2] - [x^2]}{h}$$

$$f' = \lim_{h \to 0} \frac{(x+h)(x+h) - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h \to 0} 2x+h = \boxed{2x}$$

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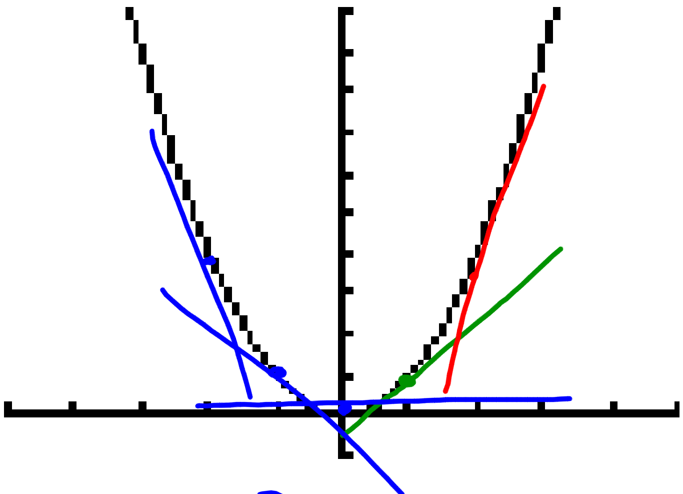

$y = x^2$

$y' = 2x$

$f' = 2x$

$f'(1) = 2(1)$

$= 2$

$f' = 2x$

$f'(1) =$

$f'(x) = 2x$

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2.) $g(x) = 6x^3$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$g' = \lim_{h\to 0} \frac{[6(x+h)^3] - [6x^3]}{h}$$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{6(x+h)^3 - 6x^3}{h}$$

$(x+h)(x+h)(x+h)$

$x^3 + 3x^2h + 3xh^2$

$$\lim_{h\to 0} \frac{6(x^3+3hx^2+3xh^2+h^3)-6x^3}{h}$$

$$\lim_{h\to 0} \frac{6x^3+18hx^2+18xh^2+6h^3-6x^3}{h}$$

$$\lim_{h\to 0} \frac{18hx^2+18xh^2+6h^3}{h}$$

$$\lim_{h\to 0} 18x^2+18xh+6h^2$$

$$18x^2$$

$$\lim_{h \to 0} \frac{[f(x+h)] - [f(x)]}{h}$$

3.) $y = x^3 + 4x^2$

$$y' = \lim_{h \to 0} \frac{[(x+h)^3 + 4(x+h)^2] - [x^3 + 4x^2]}{h}$$

ex $x^3 + 4x^2$

$$g'(x) \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$g'(x) \lim_{h \to 0} \frac{[(x+h)^3 + 4(x+h)^2] - [x^3 + 4x^2]}{h}$$

$$g'(x) \lim_{h \to 0} \frac{[x^3 + h^3 + 3xh^2 + 3hx^2 + 4(x^2 + 2xh + h^2)] - [x^3 + 4x^2]}{h}$$

$$g'(x) \lim_{h \to 0} \frac{[x^3 + h^3 + 3xh^2 + 3hx^2 + 4x^2 + 8xh + 4h^2] - (x^3 + 4x^2)}{h}$$

$$g'(x) \lim_{h \to 0} \frac{h(h^2 + 3xh + 3x^2 + 8x + 4h)}{h}$$

$$g'(x) \lim_{h \to 0} h^2 + 3xh + 3x^2 + 8x + 4h$$

$$g'(x) \lim_{h \to 0} \boxed{3x^2 + 8x}$$

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Notation

$y =$

$y' =$

y prime

$f(x) =$

$f'(x) =$

f prime of x

$y =$

$\frac{dy}{dx} =$

The derivative of y with respect to x

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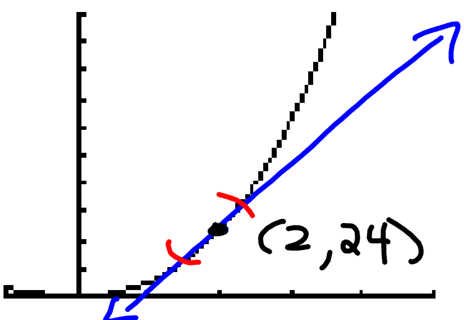
Ex2. Write the equation of the tangent line to $f(x) = x^3 + 4x^2$ at x=2.

$y - y_1 = m(x - x_1)$

$y - 24 = 28(x - 2)$



$f(2) = 2^3 + 4(2)^2$

$= 8 + 4 \cdot 4 = 24$

$f' = 3x^2 + 8x$

$f'(2) = 3(2)^2 + 8(2)$

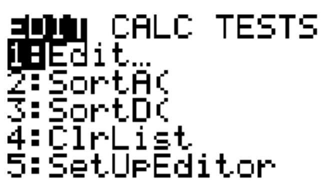
$= 3 \cdot 4 + 16$

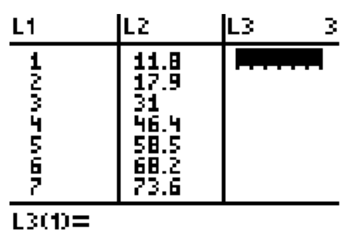
$= 12 + 16 = 28$

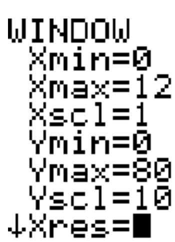
Ex3. The average monthly temperature for Minneapolis, MN is given in the table below.

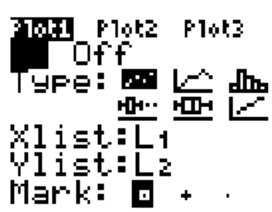
Month	Temp (F)
January	11.8
February	17.9
March	31.0
April	46.4
May	58.5
June	68.2
July	73.6
August	70.5
September	60.5
October	48.8
November	33.2
December	17.9

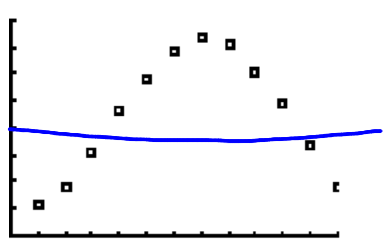
a.) Make a scatter plot of this data on your graphing calculator











b.) Estimate the derivative of the function for each month by approximating the rate of change of the temperature.

Month	Temp (F)	Derivative
January	11.8	0
February	17.9	9.6
March	31.0	14.25
April	46.4	13.75
May	58.5	10.9
June	68.2	7.55
July	73.6	1.15
August	70.5	-6.55
September	60.5	-11
October	48.8	-13.65
November	33.2	-15.45
December	17.9	-10.7

°F/month

Jan 11.8

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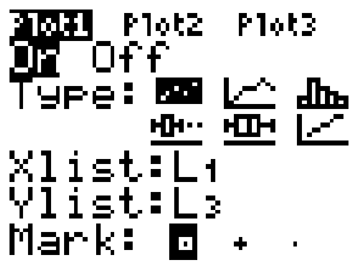
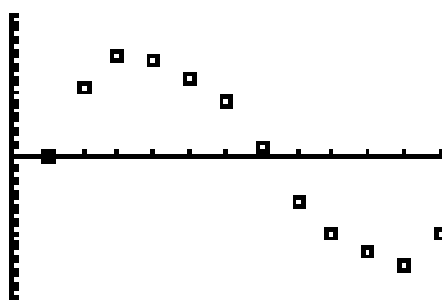
c.) Sketch a graph of the derivative for each month with pencil and paper.

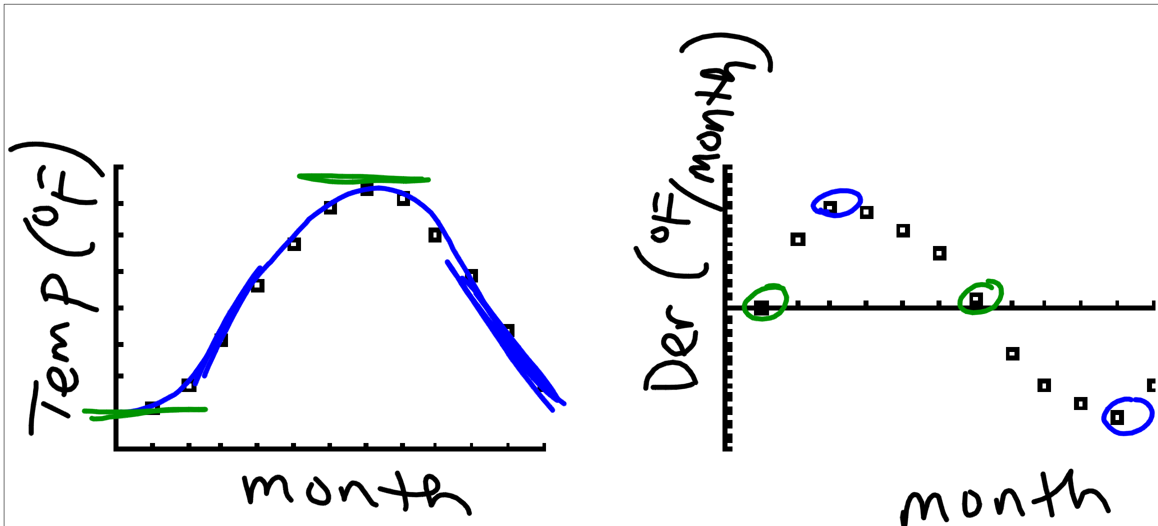
L1	L2	L3 3
1	11.8	0
2	17.9	9.6
3	31	14.25
4	46.4	13.75
5	58.5	10.9
6	68.2	7.55
7	73.6	1.15

L3(1)=0



WINDOW
Xmin=0
Xmax=12
Xscl=1
Ymin=-20
Ymax=20
Yscl=1
↓Xres=




Temp (°F)
month
Der (°F/month)
month

d.) What is the meaning of the derivative in the context of this problem?

e.) What is the units of the derivative?

f.) When the derivative is positive, what does that mean? When the derivative is negative, what does that mean?

g.) In what month(s) is the temperature changing the most rapidly?

h.) In what month(s) is the temperature changing the least rapidly?

Homework

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